THE LOWER LIMIT TO THE FIRST CRITICAL HEAT FLUX IN WEAK GRAVITATIONAL FIELDS

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ABSTRACT: Relationships are derived for the first critical heat flux in the boiling of a liquid on cylindrical and planar heaters in a weak gravitational field and on a cylindrical heater under terrestrial conditions.

1. Borishanskii and Fokin [1] have determined the lower limit to the first critical heat flux in boiling of a liquid on a planar heater under terrestrial conditions. The corresponding result for a cylindrical heater is of interest in relation to boiling on thin wires.

Consider an infinitely long horizontal cylinder of radius R with the liquid initially at rest and the initial temperature equal to the boiling point. At $t > 0$ the heater receives a heat flux q of constant density. Free convection occurs under normal conditions, so the critical q calculated without convection will be less than the true first critical flux q_*, and the result can be considered as a lower limit to the latter.

If convection is neglected, the problem is as follows:

$$c\rho \frac{\partial T}{\partial t} = \frac{\lambda}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \quad T|_{t=0} = 0, \quad \lambda\frac{\partial T}{\partial r}\Big|_{r=R} = q, \tag{1.1}$$

in which c is specific heat, ρ is the density of the liquid, λ is the thermal conductivity, t is time, and T is the difference between the temperature of the liquid and the boiling point.

The time taken for the crisis to arise is $t \gtrsim 1$ sec for some liquids of practical importance (cryogenic liquids, ethyl ether, water, etc), so the Fourier number $at/R^2 \gg 1$ (it is in the range 10^3–10^9), and so T(R, t) (difference between the heater temperature and the boiling point) [2] can be written as

$$T(R,t) = \frac{qR}{2\lambda}\ln\frac{4at}{CR^2} \quad (C = 1.781), \tag{1.2}$$

in which a is the thermal diffusivity of the liquid.

From [3] we have the radius R_1 of a bubble as a function of time as

$$\frac{dR_1}{dt} = 10\,\frac{\lambda T(R,t)}{R_1 L\rho''}, \quad R_1|_{t=0} = 0. \tag{1.3}$$

Substitution of (1.2) into (1.3) and integration gives

$$R_1 = \left(\frac{10qR}{L\rho''}\,t\ln\frac{4at}{CR^2e}\right)^{1/2} \quad \begin{pmatrix} e = 2.71828 \\ C = 1.781 \end{pmatrix}, \tag{1.4}$$

in which L is latent heat of evaporation and ρ'' is the density of the vapor.

From (1.4) we can find the time t_1 for the bubble to break away by putting $R_1(t_1) = R_0$, in which R_0 is the radius at the instant of breakaway; then

$$\tau\ln\tau = \frac{\alpha_0}{q}, \quad \tau = \frac{4at_1}{CR^2e}, \quad \alpha_0 = \frac{0.4L\rho''R_0^2}{CR^2e}. \tag{1.5}$$

To find R_0 we use [4]

$$R_0 = 0.0208\theta\left(\frac{\sigma}{(\rho-\rho'')g^\circ}\right)^{1/2}. \tag{1.6}$$

Here θ is the angle of contact, σ is the surface tension at the liquid-vapor interface, and g° is the acceleration due to gravity.

As $\alpha_0/q \gg 1$ (around 10^6), we can use an asymptotic formula [5] for the roots of (1.5):

$$\tau = \frac{\alpha_0}{q(\ln\alpha_0 - \ln q)}. \tag{1.7}$$

Consider the formula for the lower limit to q_*. If we assume that all the heat released per m^2 in time t_1 goes to produce vapor, the lower limit is

$$q = \frac{4}{3}\,\frac{\pi R_0^3 n L\rho''}{t_1}, \tag{1.8}$$

in which n is the number of interacting bubbles per m^2. We may assume that the bubbles fuse to give a bubble of ellipsoidal form; then $n \approx 1/S$, in which S is the surface area of the ellipsoid. We replace the latter by the equivalent sphere to get

$$n \approx 1/4\pi R_0^2. \tag{1.9}$$

Substitution of (1.7) and (1.9) into (1.8) gives

$$q = \alpha_0 e^{-3.333R/R_0}. \tag{1.10}$$

2. Consider the lower limit to q_* for a planar heater in a weak gravitational field. Let ΔT_0 be the superheating of the liquid, and t_0 the time needed to produce this, the latter being given [2] by

$$t_0 = \frac{\lambda^2(\Delta T_0)^2\pi}{4q^2a} = \frac{p_1}{q^2},$$

$$T(0,t) = \frac{2q}{\lambda}\left(\frac{at}{\pi}\right)^{1/2}, \quad \left(p_1 = \frac{\lambda^2(\Delta T_0)^2\pi}{4a}\right). \tag{2.1}$$

A bubble arises on the heater at $t = t_0$ and starts to grow; (1.3) and (2.1) give us for the radius that

$$\frac{dR_1}{dt} = \frac{20q\sqrt{at}}{L\rho''\sqrt{\pi}R_1}. \tag{2.2}$$

We integrate (2.2) with $R_1(t_0) = 0$ to get

$$R_1 = \frac{80^{1/2}q^{1/2}a^{1/4}}{\pi^{1/4}\sqrt{3L\rho''}}\left(t^{3/2} - t_0^{3/2}\right)^{1/2}. \tag{2.3}$$

We deduce t_1 from $R_1(t_1) = R_0$ to get

$$t_1 = \left(\frac{p_2}{q} + t_0^{3/2}\right)^{2/3}, \quad p_2 = \frac{3\sqrt{\pi}R_0^2L\rho''}{80}, \quad R_0 = \frac{R_{0n}}{n^{1/3.5}}. \tag{2.4}$$

Here R_0 is defined as in [6], with R_{0n} calculated from (1.6); $n = g/g^\circ$, $g^\circ = 9.81$ m/sec^2 is the acceleration due to gravity at the earth's surface, and g is the actual gravitational acceleration.

The argument of section 1 gives the lower limit for q_* as

$$q = \frac{\pi R_0 L\rho''}{3t_1}. \tag{2.5}$$

We substitute for t_1 from (2.4) to get

$$p_2 s^4 - \left(\frac{\pi R_0 L\rho''}{3}\right)^{3/2}s^3 + p_1^{3/2} = 0, \quad q = s^2.$$

This equation can be solved graphically.

3. Consider the lower limit to q_* for a horizontal infinitely long cylindrical heater (radius R) in a weak gravitational field. We find t_0 from (1.2):

$$t_0 = \frac{CR^2}{4a}\exp\left(\frac{2\lambda\Delta T_0}{qR}\right). \tag{3.1}$$

We solve (1.3) with $R_1(t_0) = 0$ to get

$$R_1 = \left[\frac{10qR}{L\rho''}\left(t\ln\frac{4at}{CR^2e} - t_0\ln\frac{4at_0}{CR^2e}\right)\right]^{1/2}. \tag{3.2}$$

We deduce t_1 from $R_1(t_1) = R_0$ via the method of [5]:

Table 1

n	Calculated		Experimental	
	$q_* 10^{-4}$ W/m²	$\frac{q_*}{q_{*n}}$	$q_* 10^{-4}$ W/m²	$\frac{q_*}{q_{*n}}$
1.0	6.6	1.0	15.6	1.0
0.5	2.6	0.40	13.1	0.84
0.3	2.5	0.38	11.6	0.74
0.1	2.2	0.33	8.6	0.56
0.04	2.15	0.33	7.0	0.45
0.01	2.0	0.30	4.9	0.31
0.005	1.7	0.26	4.1	0.27

$$\tau = \frac{\alpha_0 + \beta_1 q}{q \ln (\alpha_0/q + \beta_1)} \quad \left(\tau = \frac{4at_1}{CR^2e}\right),$$

$$\alpha_0 = \frac{0.4L\rho'' R_0^2 a}{CR^3 e}, \quad \beta_1 = \frac{4at_0}{CR^2e} \ln \frac{4at_0}{CR^2e}. \tag{3.3}$$

We substitute (3.1) and (3.3) into (1.8) to get

$$\beta_2 = \frac{\beta_0 + (\exp(c_0/q) - 1)(c_0 - q)}{\ln[\beta_0/q + (\exp(c_0/q) - 1)(c_0/q - 1)]},$$

$$c_0 = \frac{2\lambda \Delta T_0}{R}, \quad \beta_2 = \frac{4L\rho'' a R_0}{3CR^2e}, \quad \beta_0 \equiv \alpha_0. \tag{3.4}$$

This equation can be solved graphically.

For weak fields, R_0 is deduced from (2.4). The table gives results for q_* from theory and experiment for liquid oxygen with $\Delta T_0 \approx 10°$. The calculation for g° was performed via (1.10) and for g via (3.4).

If we replace (2.4) for weak fields by a published relation:

$$R_0 = R_{0n} n^{-1/2} \quad \text{for} \quad n > 0.1$$

and

$$R_0 = R_{0n} n^{-1/3} \quad \text{for} \quad n < 0.1,$$

the results for q_* in the table remain unchanged.

These experimental results were obtained at the Institute of Low-Temperature Physics Technology, Academy of Sciences of the Ukrainian SSR, with simulation of low-gravity conditions for liquid oxygen in a magnetic field [8]. We used a platinum wire 0.05 mm in diameter. The results for q_* with $0.01 < n \leq 1$ are closely described by the Kutateladze-Borishanskii-Zubra formula. This formula also agrees satisfactorily with experiments on water and liquid nitrogen [6, 9].

Table 1 shows that the calculated q_* are of the correct order and represent the lower limit to the first critical flux. The calculated and experimental q_*/q_{*n} are virtually the same for small a (around 10^{-2}). The results are applicable for $10^{-3} < n \leq 1$.

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